Histogram construction example

This is an example of normality test implementation based on Pearson's (χ^2) criterion. A data set consists of $N = 1000$ points and is generated by `randn` function. The number of histogram bins is $\log_2 N = 10$.

The first step is the estimation of mathematical expectation μ and variance σ to build theoretical normal distribution. The next formula are used:

$$\mu = \frac{1}{N}\sum_i X_i;\ \sigma = \sqrt{\frac{\sum_i (X_i - \mu)^2}{N-1}}$$

For the input data set $\mu = -0.0001$, $\sigma = 0.9728$

The second step is the construction of empirical histogram (that gives us observed frequencies O_i) and calculation of theoretical histogram using μ and σ values obtained above The next formula is used for E_i (expected frequencies calculation):

$$E_i = N\int_{x_{i,\min}}^{x_{i,\max}} \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx$$

Obtained E_i and O_i values are given in Table 1, empirical histogram is shown at Figure 1

The third step is applying of Pearson's χ^2 test using the next relations:

$$\chi^2_{\text{emp}} = \sum_i \frac{(O_i - E_i)^2}{E_i} = 5.383$$

$$\chi^2_{\alpha,f}\,(\alpha = 0.95, f = n_{\text{bins}} - 1 = 9) = 16.919$$

$\chi^2_{\text{emp}} \le \chi^2_{\text{crit}}$ and the empirical distribution can be considered normal.

Table 1: Observed and expected histogram frequencies

$x_{i,\min}$	$x_{i,\max}$	O_i	E_i	$(O_i - E_i)^2/E_i$
$-\infty$	-2.389	11.0	7.0	2.250
-2.389	-1.774	26.0	27.1	0.044
-1.774	-1.159	84.0	82.7	0.020
-1.159	-0.543	156.0	171.4	1.388
-0.543	0.072	245.0	241.2	0.059
0.072	0.687	242.0	230.6	0.567
0.687	1.303	144.0	149.7	0.214
1.303	1.918	71.0	66.0	0.386
1.918	2.533	17.0	19.7	0.377
2.533	$+\infty$	4.0	4.6	0.080

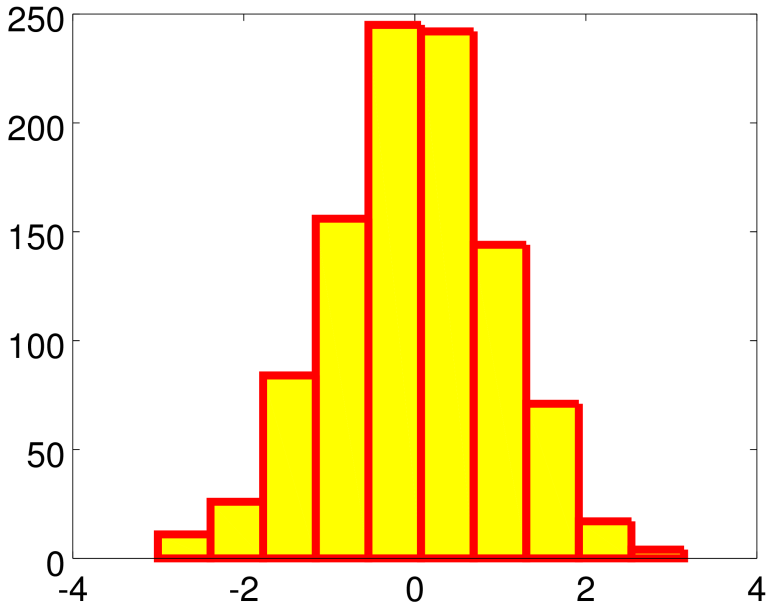

Figure 1: Empirical histogram

Input data set

Table 2: Input data set

X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i
-0.5717	0.4181	-0.1321	0.9851	-0.5101	-0.1682	0.0752	1.7556
0.4949	-1.0727	-0.4587	1.3097	-0.3692	0.2458	-0.6881	-0.3692
-0.0038	-0.0333	-0.1280	0.6885	1.1769	0.1347	1.1182	1.1756
1.2388	-0.6594	-0.2426	1.8452	0.0736	1.2859	-0.9324	-0.1836
-0.6510	0.2451	0.1115	-0.0148	1.1030	-0.5156	-0.8164	0.7090
0.6431	0.9501	0.5640	-0.0819	0.1604	0.1318	-0.4469	0.1619
0.7392	1.2146	0.9774	-0.7970	0.2805	0.2412	1.2417	-0.0594
0.1440	0.8331	0.1833	-0.9279	-0.8808	0.0026	0.5999	0.4871
-0.7379	1.2817	-0.5194	0.3559	0.3463	-0.8642	0.9988	1.1380
2.3167	0.7453	-0.3531	0.6858	2.2008	1.5666	-0.5189	0.3492
-0.5436	-1.1865	-0.0009	-0.0111	1.3068	-0.9186	0.1989	-1.2377
-1.3260	1.7221	-0.2827	-0.1469	1.0118	-1.7920	-1.1893	-0.5237
-0.3200	-0.4972	1.2330	-0.1478	1.7617	0.8002	-0.2794	0.1352
1.0512	-0.8892	0.5478	1.3320	1.5431	1.8743	0.1385	-0.0769
1.1138	0.4214	-1.4898	0.1059	2.0948	-0.1094	1.3047	-0.2541
0.1342	-1.8712	0.8723	0.1523	-0.4136	0.3332	0.5997	1.8046
-0.0832	-0.2254	1.0267	-0.4399	1.4023	1.1138	-0.2064	0.8213
-0.7988	-0.4502	-0.2288	1.3764	-1.9790	0.5130	-0.2327	-0.0819
0.3346	-1.3322	0.7026	0.5671	-1.6290	0.7120	0.3327	0.4049
X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i

X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i
-1.2145	0.6306	0.1986	-0.4972	0.2578	-0.0853	0.3171	-0.1716
0.0917	0.3041	-0.8690	-0.0440	-0.4387	-0.1526	0.3634	0.2205
1.6438	-1.2317	-0.0150	-3.0045	-0.1805	-0.8068	1.3264	0.0021
0.5022	0.4020	1.0367	-1.6251	0.4023	-2.0540	0.3167	0.4475
1.3488	1.0995	0.1355	0.4270	-0.1913	0.2191	1.0134	0.5209
-2.4575	-0.9634	0.1737	-0.5793	0.0931	-1.2659	0.2741	0.2776
-0.7136	-1.8544	0.0433	0.4989	-0.2871	0.2973	-0.0351	1.6390
-1.9552	-0.0142	-2.2294	1.3220	-2.5600	0.4487	-1.1271	0.5064
0.5116	1.3226	0.6859	0.6408	-0.2703	0.4102	-0.0058	0.5627
0.0176	-1.5219	0.7390	0.0187	-0.7633	-0.2965	-0.0233	-1.2961
1.3072	-0.1341	1.2636	0.2339	-0.3342	-0.1510	1.7391	-0.2758
-0.8013	0.7077	2.4412	-0.4874	2.0030	0.7440	-1.3994	0.7081
0.9010	1.1683	-0.0661	-0.4017	0.0904	-1.0458	-0.8944	0.5421
-0.0699	-0.0396	1.3101	0.0969	-0.1888	-0.8493	-0.0126	-0.4008
1.6353	-0.8938	-0.0394	0.6588	0.4259	-1.4058	-0.7287	0.4641
0.5777	1.1556	1.0548	-0.3273	1.0937	-1.2110	-0.3351	0.4251
0.9642	0.3773	-1.7588	0.6150	0.4584	-0.7890	-0.3377	0.5501
-0.9370	-0.8158	1.0407	0.3495	-1.2566	0.6815	-1.0729	0.3722
0.6506	0.2352	2.9323	-2.5114	1.7850	-0.7870	-0.8950	-0.1334
-1.3901	-0.9693	1.2688	-0.3706	0.5752	-0.5349	0.4743	0.6607
1.5272	-0.8751	-0.2191	-0.7850	1.0999	-0.0051	0.2447	0.2135
0.2955	0.7511	-1.2452	-0.9981	-0.7034	-0.7757	1.1165	0.5507
-0.8979	0.9403	1.0600	-2.7504	0.8990	1.0075	0.2449	-0.4563
0.8082	0.1908	1.0676	-2.2506	-0.2643	0.5512	0.3407	0.1049
-0.5651	-0.6147	-0.7294	1.4682	0.5125	0.8209	0.5102	1.6372
0.3543	0.1176	0.8522	-0.2724	1.9747	0.6745	-0.6022	-1.0966
0.4963	1.7885	-0.0345	-1.6224	-0.5002	0.4556	0.4114	-1.6642
0.9273	-0.4415	1.3121	-1.0061	0.1769	-0.6894	-1.9525	-1.1919
-0.9145	0.3093	0.2758	1.7721	-0.9143	1.5650	-0.4914	-0.4890
-0.9941	0.1546	0.4061	0.2061	-0.7840	-1.2263	0.4743	1.1662
0.4729	-0.0066	-1.2733	0.8645	-0.8708	1.9204	1.9891	0.7891
-2.0467	-1.4711	-0.2725	0.3291	-0.0720	-0.4977	-0.9260	-0.5282
-0.1137	-0.6819	-0.6987	-0.0086	-0.7401	-0.8445	0.2648	0.5221
0.8240	1.1453	0.6771	0.7610	-1.2991	0.3967	0.4863	1.1265
0.8243	-1.0223	-0.9737	1.5419	-0.6147	-0.1733	-1.6015	-0.3674
0.2090	0.6368	0.6626	-0.6336	0.5279	-1.0689	0.1258	-0.1462
-1.0034	1.3530	2.1183	-2.8630	-1.6140	-0.0690	-0.6456	-1.6719
0.0097	0.1236	-0.9410	-0.8512	1.3968	0.2894	0.0755	-0.0387
1.6099	-0.5494	0.7058	-0.3453	-0.9405	0.8653	-0.7944	-1.1107
-0.5331	-0.7784	1.0386	-1.1337	-2.1599	0.4232	-0.2981	0.6705
-0.0909	-2.2152	-0.1244	-1.0197	1.1794	-0.1864	-0.0109	-0.0478
-0.0427	1.0914	-0.0782	-1.6736	-0.1115	-0.6998	-0.2679	-1.1121
0.5376	-0.2965	-0.5969	0.6049	0.0206	-0.7153	1.1374	-2.2189
0.9198	1.4782	-0.8870	-0.5686	1.5169	-0.3817	-1.1199	1.0029
0.6081	0.6416	-0.1144	-0.4817	-0.4879	0.2335	0.6082	0.7275
X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i

X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i
1.0613	0.8420	1.5627	0.3713	-0.0926	-2.6674	-1.1555	-0.0945
-0.3084	-1.3111	-1.1940	-2.6731	0.7295	-0.6873	0.7473	0.1866
-0.0938	0.6544	1.4128	0.8132	0.1017	-0.1938	1.7108	-1.1011
1.0930	0.1697	-1.7895	0.1301	0.5691	-0.5205	0.4750	2.5653
-0.5685	-0.3898	0.0395	-1.5170	-0.5195	0.6319	-0.3249	0.5075
0.6705	0.0699	0.1162	-1.3829	0.7619	-1.9522	0.3183	0.6170
-1.1421	0.8569	0.6007	-1.3223	-0.6659	1.7612	-0.2255	0.6309
-1.2848	-0.3261	-0.4341	-0.2640	-0.3444	0.1775	-0.8415	0.9485
1.4607	-1.2279	0.3171	-0.3210	0.6147	-1.3358	-0.4783	-0.1707
0.2166	-0.4858	1.8486	-0.7873	-0.5421	-0.7500	-0.4337	0.9026
-0.6848	-0.2099	1.5455	-1.0352	0.1708	-0.1750	-0.0432	-0.6901
-1.1988	-0.3113	1.8432	1.1934	1.2379	-0.3844	0.3134	-1.0208
-1.7238	0.7012	-0.5932	1.2833	0.3926	0.3507	0.5280	0.9919
0.3291	-0.0150	-0.4706	0.4562	0.1926	-0.1136	0.0055	0.3628
-2.4695	0.5635	-0.9910	0.7032	0.6906	-0.7614	-0.2710	-1.4285
-0.7820	-1.5073	1.0379	0.0827	0.9170	1.3233	0.7928	-2.1721
1.2346	-0.3820	-2.3228	1.5126	0.2212	-0.8461	-0.6575	0.9389
0.5555	2.1807	0.2251	0.1270	0.0640	-0.7212	1.4241	-1.4300
-0.1258	-0.3580	0.0278	-0.3539	0.5339	1.1130	0.7459	0.4558
-0.1073	0.7298	-0.3047	-1.6103	-1.1858	1.2571	0.2986	0.6506
-1.3502	-0.6574	-0.2318	0.2847	-0.9709	1.1871	1.0470	1.9197
-1.3746	0.8528	1.5327	0.7660	-1.2592	-0.7526	-1.6406	0.1841
0.9805	1.0142	-0.9710	1.6203	-0.3460	1.2928	-0.1928	0.4090
0.6169	0.3725	0.3383	-0.1612	0.1953	1.3480	-0.7035	-0.2998
0.5223	-0.9379	1.6361	-0.3163	1.2131	-0.3467	-0.4309	0.5433
-0.5163	-0.7068	0.1120	0.5477	1.3359	-0.8944	0.3444	-0.4816
-0.8110	-0.0790	-1.6130	-1.3083	0.0616	-0.0695	-0.2045	-0.2415
-0.2999	-1.9083	2.5416	-0.2284	1.3230	-0.4537	-1.3511	-1.4550
0.6184	0.5923	-1.1338	0.1929	0.5961	0.7560	-1.1079	1.3355
-0.6813	-0.4402	-1.2690	-1.0628	-1.1962	1.3601	-1.1058	0.5134
-0.0433	-1.6204	-0.2951	-0.4444	-0.0626	-1.6359	0.7841	0.2359
-0.7228	0.0660	-0.6955	1.0920	0.2448	0.8845	-0.6917	-1.1573
0.0518	0.0467	1.3309	-1.1779	-1.4557	1.3563	-1.9419	-0.1447
-0.1999	0.4158	-0.3323	-1.8589	0.2022	-1.0320	-0.5146	0.9922
1.1663	-0.1098	0.6407	0.0726	-2.2985	-0.2945	-1.9991	-1.2307
-1.0361	-0.6099	-0.6593	-1.3140	-0.7579	-0.1697	1.1510	0.7616
1.9756	1.3736	0.7224	1.4566	-0.1384	0.4627	1.7364	0.3544
-1.4250	0.5366	0.8390	-0.6579	-2.7981	-1.8301	-1.1587	-1.4824
0.7291	-1.5981	-0.5650	-0.5630	-1.5618	0.1869	0.0367	0.9995
1.3586	1.8870	-1.3345	-1.0423	0.0324	0.6956	1.1211	-0.8301
-0.1282	0.8280	-1.4013	-1.2966	3.1484	0.2002	-0.1178	-0.2070
1.2812	0.5300	0.4523	-0.3817	-0.3235	0.3677	-0.5555	0.1984
1.0760	1.5087	1.1700	0.0769	0.1063	0.6067	0.9068	-0.1686
-0.5343	-0.3749	1.1397	-1.0402	-0.9668	-0.2195	2.3367	1.0353
0.2879	0.8734	0.8599	-0.2410	0.8161	-1.3077	1.1718	-1.2683
X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i

X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i
0.6425	-1.4282	1.4193	-0.5178	-0.9781	-1.5716	-0.9520	0.3815
1.0139	1.8943	-0.1213	0.1242	0.3501	0.0565	-0.0771	-0.8283
-0.0610	-1.1422	-0.2137	-0.2959	-0.7038	0.2673	-0.9659	-1.9282
2.3888	0.2926	0.9932	2.0536	-0.4970	-0.4687	0.0610	-0.2394
0.6263	0.2925	-0.0267	-0.2917	0.0481	-0.2216	-1.2524	0.0010
-0.9839	-1.3328	0.0914	-1.4046	-0.6745	-0.4228	-0.7347	0.3504
0.5610	0.3455	0.0074	-0.2320	1.4486	-0.6304	-1.8243	-0.8901
0.1862	1.1391	-0.6249	0.4810	0.3913	-0.1071	-0.3724	0.7197
0.8821	1.7301	-0.7138	-0.2305	-0.1730	-1.1478	0.1680	-1.0767
-0.1373	1.4098	0.3318	0.5108	1.9500	-1.3086	0.1267	1.0188
-0.1636	-1.6504	-0.1672	0.9406	-0.3259	0.5556	0.6344	-0.4314
-1.5992	-1.6527	-0.2261	0.1733	0.5348	-1.0505	0.1456	0.3934
-1.2034	-0.9409	-0.4332	0.6801	-1.4637	-0.3210	-0.4216	1.5558
-2.6699	-0.4288	0.3924	2.2930	0.7502	-0.8151	-0.8797	0.3282
-0.3684	-0.2417	-1.1045	-0.2062	-0.6417	-0.7139	-2.1508	-0.3488
0.4650	0.7478	-0.7493	0.6593	1.0350	-1.1887	0.1701	0.0362
X_i	X_i	X_i	X_i	X_i	X_i	X_i	X_i